

Exercise 26

Use a linear approximation (or differentials) to estimate the given number.

$$\sqrt{100.5}$$

Solution

Compute the derivative of $y = \sqrt{x}$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sqrt{x} \\ &= \frac{d}{dx} (x^{1/2}) \\ &= \frac{1}{2} x^{-1/2}\end{aligned}$$

Consequently, the differential of $y = \sqrt{x}$ is

$$dy = \frac{1}{2} x^{-1/2} dx.$$

In order to estimate $\sqrt{100.5}$, set $x = 100$ and $dx = 0.5$.

$$dy = \frac{1}{2} (100)^{-1/2} (0.5) = \frac{1}{40} = 0.025$$

Note that dy here is the vertical distance from the function's actual value at $x = 100$ to the linear approximation's value at $x = 100.5$.

$$\sqrt[3]{100.5} \approx \sqrt{100} + \frac{1}{40} = 10.025$$